

$$V = \frac{5\sqrt{2}}{96}\pi^4 - \frac{\sqrt{2}}{2}\pi^2$$